Indian Statistical Institute, Bangalore

B. Math. Third Year

Second Semester - Differential Equation

Final Exam Duration: 3 hours Date : April 25, 2016

Max Marks: 50

Section I: Answer any four and each question carries 6 marks.

1. Solve $y' + P(x)y = Q(x)y^n$, for $n = 0, 1, 2, \cdots$ where P and Q are continuous functions.

2. Let y be a solution of a homogeneous 2nd order linear differential equation on $[a, b]$. Prove that $\{t \in [a, b] \mid y(t) = 0\}$ is a finite set or $y(t) = 0$ for all $t \in [a, b]$.

3. (a) Does $y' = f(x, y)$, $y(0) = 0$ have unique solution for a continuous function f on $[-1, 1] \times [-1, 1]$? Justify your answer (*Marks: 3*).

   (b) Find the differential equation satisfied by the family of curves $\{ax + bx^2 \mid a, b \in \mathbb{R}\}$.

4. Solve $4y'' - x^2y + 3y = 0$.

5. Show that any solution of $y'' + xy = 0$ is $y = \sqrt{x}[aJ_{\frac{1}{3}}(\frac{2}{3}x^{\frac{3}{2}}) + bJ_{\frac{-1}{3}}(\frac{2}{3}x^{\frac{3}{2}})]$.

6. Find all solutions u of the 2-dimensional heat equation that satisfy the homogeneous Dirichlet condition and are of the form $u(x, y, t) = F(x)G(y)H(t)$.

Section II: Answer any two and each question carries 13 marks.

1. (a) Solve $xy'' = y' + (y')^3$ (*Marks: 6*).

   (b) Solve $2x^2y'' + x(2x + 1)y' - y = 0$ by Frobenius method.

2. (a) Find all polynomial solutions of $y'' - 2xy' + 12y = 0$.

   (b) Find the general solution of $x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$ near $x = 0$ where a, b, c are constants and $c \notin \mathbb{Z}$. (*Marks: 7*).

3. (a) State and prove the mean value property for harmonic functions on open subsets of $\mathbb{R}^2$.

   (b) Solve $(3y - 2u)u_x + (u - 3x)u_y = 2x - y$, $u(s, s) = 0$ (*Marks: 7*).